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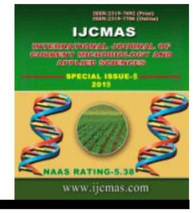


**Original Research Article**

# Comparison of Some Ratio Estimators Using Linear Transformation

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Ratio estimator, Linear transformation, Transformed ratio estimator, Precision

**A B S T R A C T**

Various authors have proposed different version of linear transformation of the auxiliary variable (x) in sample surveys to reduce the bias and mean square error (MSE) of usual ratio estimator. Notable among them are Mohanty and Das (1971), Reddy (1974), Srivenkatramana (1978), Das and Tripathi (1980), Sisodia and Dwivedi (1981), Singh and Kakran (1993), Mohanty and Sahoo (1995), Upadhyay and Singh (1999) etc. A comprehensive review of transformed ratio estimator is presented in this paper. Moreover, when a prior value of y– intercept in the simple linear regression of y on x is available, a linear transformation of study variable y is suggested to find out another modified transformed ratio estimator. Properties of proposed modified transformed ratio estimator are studied. An empirical study with some real populations is also carried out to highlight the precision of transformed ratio estimator.

## Introduction

The precision of the usual ratio estimator has been improved by developing transformed ratio estimators using linear transformation of the auxiliary variable X linearly related to the study variable Y in the past [Mohanty and Das (1971), Reddy (1974), Srivenkatramana (1978), Das and Tripathi (1980), Sisodia and Dwivedi (1981), Singh and Kakran (1993), Mohanty and Sahoo (1995), Upadhyay and Singh (1999) etc.]

In fact, such linear transformation is generally aimed to enable that the linear regression line of Y on transformed auxiliary variate passes through the origin because in this situation the ratio estimator becomes unbiased and becomes as efficient as the usual regression estimator.

## Linear transformation and transformed ratio estimator

Let $U=(U_1,U_2,U_3,.....,U_N)$ be a finite population comprised of N identifiable units. Associated with the unit $U_i$ is the two numbers $(y_i,x_i)$ representing the values of $Y_i$, the study variate, and $X_i$, the auxiliary variate (i=1,2,…, N). We assume that $X_i$'s are known for all i=1, 2,…, N. We also assume that Y and X are linearly related by the following regression function.

$$Y_i=\alpha+\beta X_i+e_i, \quad (i = 1, 2, 3…, N) \qquad … (1.1)$$

where α and β are Y –intercept and regression coefficient of Y on X, respectively $e_i$ is the error term assumed to have $E(e_i)=0$ and $V(e_i)=\sigma^2$, where E stands for mathematical expectation, It is also assumed that Y and X are positively correlated. It is well known that if the regression line of Y on X passes through the origin, the usual ratio estimator is unbiased and is equally efficient as regression estimator. This notion has resulted in the past to use linear transformation of the auxiliary variable in order to make the regression line of Y on transformed variate X passes through the origin, and accordingly various efficient transformed ratio estimators have been developed by various authors.

However, if a prior value of Y- intercept $\alpha$ is known, a linear transformation of the study variate Y could be easily done to make the linear regression line of the transformed variate Y on X pass through the origin. This idea has been explored in the present investigation to develop an efficient transformed ratio estimator by transforming the character under study Y. Another simple transformation of X using its minimum and maximum values has been done and a new estimator developed.

## Transformed ratio estimator by transforming the study variate Y

Let $a_0$ be a prior known value of Y-intercept $\beta$ in the equation (1.1). The transformation of Y then follows as

$$Y_i - a_0 = \beta X_i + e_i, \ (i = 1, 2, 3 \ldots N)$$

denoting $Y_i - a_0 = Z_i$, a transformed study variate, the above equation can be written as-

$$Z_i = \beta X_i + e_i \ (i=1, 2, 3\ldots N) \quad \ldots(1.1.1).$$

Consider that a random sample of size n is drawn from finite population U by simple random sampling without replacement. We define the following

$\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$, sample mean of y

$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$, sample mean of x

$\bar{z} = \bar{y} - a_0$, sample mean of z

$\bar{Y} = \sum_{i=1}^{n} \frac{Y_i}{N}$, the population mean of Y

$\bar{Z} = \sum_{i=1}^{N} \left( \frac{Y_i - a_0}{N} \right) = \sum_{i=1}^{N} \frac{z_i}{N}$, the population mean of Z

$\bar{X} = \sum_{i=1}^{n} \frac{X_i}{N}$, the population mean of X

$S_y^2 = \sum_{i=1}^{N} \frac{(Y_i - \bar{Y})^2}{(N-1)}$, the mean square for Y in the population

$S_x^2 = \sum_{i=1}^{N} \frac{(X_i - \bar{X})^2}{(N-1)}$, the mean square for X in the population

$R = \frac{\bar{Y}}{\bar{X}}$, the ratio of population means

$R_1 = \frac{\bar{Z}}{\bar{X}}$, the ratio of population means of Z to X

$S_{yx} = \sum_{i=1}^{N} \frac{(Y_i - \bar{Y})(X_i - \bar{X})}{(N-1)}$, the covariance between Y and X in the population

$\rho = \frac{S_{yx}}{S_y S_x}$, the correlation coefficient between Y and X in the population

$C_y = \frac{S_y}{\bar{Y}}$, the coefficient of variation in Y

$C_x = \frac{S_x}{\bar{X}}$, the coefficient of variation in X

The usual ratio estimator of the population mean $\bar{Y}$ is given by

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}.\bar{X} \qquad \text{... (1.1.2)}$$

The usual ratio estimator is modified by applying linear transformation of the study variate. Therefore a transformed ratio estimator of the population mean $\bar{Y}$ is proposed as

$$\bar{y}_{pr(1)} = \frac{\bar{y}\bar{X} + a_0(\bar{x} - \bar{X})}{\bar{x}} \qquad \text{... (1.1.3)}$$

Alternatively, it can also be written as

$$\bar{y}_{pr(1)} = \frac{\bar{z}\bar{X} + a_0\bar{X}}{\bar{x}} \qquad \text{... (1.1.4)}$$

Obviously the proposed transformed ratio estimator $\bar{y}_{pr(1)}$ is in general not an unbiased estimator of $\bar{Y}$. We shall derive its bias and mean square error (MSE).

The bias in $\bar{y}_{pr(1)}$ is given by

$$B(\bar{y}_{pr(1)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{\bar{Z}S_x^2}{\bar{X}^2} - \frac{S_{yx}}{\bar{X}}\right]$$

$$= \left(\frac{1}{n} - \frac{1}{N}\right)\left[\bar{Z}C_x^2 - \bar{Y}\rho C_x C_y\right] \qquad \text{...(1.1.5)}$$

MSE of the proposed transformed ratio estimator $\bar{y}_{pr(1)}$ is derived as follows:

$$MSE(\bar{y}_{pr(1)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left[C_y^2 + R_1'^2 C_x^2 - 2R'_1\rho C_y C_x\right] \qquad \text{...(1.1.6)}$$

where $R_1' = \frac{(\bar{Y} - a_0)}{\bar{Y}} = \frac{\bar{Z}}{\bar{Y}}$

The above results can be summarized in the following theorem.

**Theorem1.1:** Under the linear transformation of the study variate, the modified transformed ratio estimator of the population mean

$$\bar{y}_{pr(1)} = \frac{\bar{y}.\bar{x} + a_0(\bar{x} - \bar{X})}{\bar{x}}$$

is an almost unbiased estimator with its MSE as-

$$MSE\left(\bar{y}_{pr(1)}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}^2\left[C_y^2+R'^2_1 C_x^2-2R'_1\rho C_y C_x\right]$$

where $R_1'=\frac{\left(\bar{Y}-a_0\right)}{\bar{X}}$

**Theorem1.2:** Under the linear transformation of the study variate Y, if $a_0=\alpha$, i.e. the priori value is exactly equal to $\alpha$, then proposed transformed ratio estimator $\bar{y}_{pr(1)}=\frac{\bar{y}.\bar{x}+a_0\left(\bar{x}-\bar{X}\right)}{\bar{x}}$ is an unbiased estimator of the population mean $\bar{Y}$ with its MSE exactly equal to the MSE of the usual biased regression estimator $\bar{y}_{lr}$, i.e.

$$MSE\left(\bar{y}_{pr(1)}\right)=MSE\left(\bar{y}_{lr}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2\left(1-\rho^2\right)$$

**Proof**

$$MSE\left(\bar{y}_{pr(1)}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_y^2+\left(\frac{\bar{Y}-\left(\bar{Y}-\beta\bar{X}\right)}{\bar{X}}\right)^2 S_x^2-2\left(\frac{\bar{Y}-\left(\bar{Y}-\beta\bar{X}\right)}{\bar{X}}\right).\rho S_y S_x\right]$$

$$=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_y^2+\beta^2 S_x^2-2\beta\rho S_y S_x\right]$$

Since $\beta=\rho.\frac{S_y}{S_x}$ we have the above expression as follows:

$$MSE\left(\bar{y}_{pr(1)}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_y^2+\rho S_y^2-2\rho S_y^2\right]$$

$$=\left(\frac{1}{n}-\frac{1}{N}\right)\left[S_y^2\left(1-\rho^2\right)\right]$$

which is exactly equal to the MSE of the usual biased regression estimator $\bar{y}_{lr}$. Hence, the theorem is proved.

Under the linear transformation of the study variate y, if $a_0$=a, then the regression line of transformed study variate Z on X passes exactly through the origin and therefore, bias of $\bar{y}_{pr(1)}$ will be exactly equal to zero. This result is also obvious from the facts given on page 161 of Cochran (1963). Hence, the proposed transformed ratio estimator is an unbiased estimator.

The MSE of the usual biased regression estimator $\bar{y}_{lr}$ (Cochran, 1963) is given by

$$MSE\left(\bar{y}_{pr(1)}\right)=MSE\left(\bar{y}_{lr}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2\left(1-\rho^2\right)$$

If $a_0=\alpha$ we know that $\alpha=\bar{Y}-\beta\bar{X}$

Replacing $a_0$ by $\alpha=\bar{Y}-\beta\bar{X}$, we get

## Comparison of the estimators

The relative efficiency of the proposed transformed ratio estimator in comparison to the simple sample mean $\bar{y}$ and the usual ratio estimator will be determined in this section.

The variance of $\bar{y}$ is given by

$$V\left(\bar{y}\right)=\theta S_y^2 \qquad \ldots(2.1.1)$$

where $\theta=\left(\frac{1}{n}-\frac{1}{N}\right)$

The MSE of usual ratio estimator $\bar{y}_r$ is given by

$$MSE\left(\bar{y}_r\right)=\theta\left[S_y^2+R^2S_x^2-2R\rho S_y S_x\right] \qquad \ldots(2.1.2)$$

The MSE of the proposed transformed ratio estimators $\bar{y}_{pr(1)}$ is given by

$$MSE(\bar{y}_{pr(1)}) = \theta[S_y^2 + R_1^2 S_x^2 - 2R_1\rho S_y S_x] \quad (2.1.3)$$

### Comparison of $\bar{y}_{pr(1)}$ with $\bar{y}_r$ and $\bar{y}$

Comparing $MSE(\bar{y}_{pr(1)})$ with $V(\bar{y})$, we get easily that $MSE(\bar{y}_{pr(1)})$ will be more efficient than $\bar{y}$ if

$$\rho > \frac{1}{2}\frac{\bar{Z}}{\bar{Y}}.\frac{C_x}{C_y} = \frac{\bar{Y} - a_0}{\bar{Y}}\frac{C_x}{C_y} = A_1 \quad \text{(Say)}$$

…(2.1.1.1)

If $a_0$ is positive, then $\frac{\bar{Z}}{\bar{Y}} < 1$, the range of ρ would be increased for getting more precise estimate of $\bar{Y}$ even for lower values of ρ.

Comparing the equation (2.1.2) and (2.1.3), we get $\bar{y}_{pr(1)}$ will be more efficient than $\bar{y}_r$ if

$$\theta[S_y^2 + R^2 S_x^2 - 2R\rho S_y S_x] > \theta[S_y^2 + R_1^2 S_x^2 - 2R_1\rho S_y S_x]$$

or

$$(R^2 - R_1^2)S_x^2 > 2(R - R_1)\rho S_y S_x$$

or

$$(R_1 - R)[(R_1 + R)S_x - 2\rho S_y] < 0$$

…(2.1.1.2)

It is also known that $\bar{y}_r$ will be more efficient than if $\bar{y}$ if

$$\rho > \frac{1}{2}\frac{C_x}{C_y} = A_2 \quad \text{(say)}$$

…(2.1.1.3)

Now, $(R_1\text{-}R)$ may be negative or positive depending upon the sign of intercepts $a_0$

**Case I:** if $a_0>0$

In this case $R_1\text{-}R<0$. Therefore, the inequality will (2.1.1.2) hold true only if

$$(R' + R)S_x - 2\rho S_y > 0$$

or

$$\rho < \frac{1}{2}\left(1 + \frac{\bar{Z}}{\bar{X}}\right)\frac{C_x}{C_y} = A_3$$

(say) …(2.1.1.4)

Clearly, $A_3$ is greater than $A_2$. Also, $A_1<A_2$ Obviously, $A_3$ is upper bound for ρ below which $\bar{y}_{pr(1)}$ will be more efficient than $\bar{y}_r$. Hence, $\bar{y}_{pr(1)}$ will be more efficient than $\bar{y}_r$ and $\bar{y}$ both as long as ρ lies between $A_1$ and $A_3$.

**Case II:** If $a_0<0$

In this case $R_1\text{-}R>0$ and therefore the inequality (2.1.1.2) will hold true only if

$$(R' + R)S_x - 2\rho S_y < 0$$

Writing $\bar{Z}'$ for $\bar{Z}$ in case $a_0<0$, we have

$$\rho > \frac{1}{2}\left(1 + \frac{\bar{Z}'}{\bar{Y}}\right)\frac{C_x}{C_y} = (A_2 + A_1)$$

$= A_4$

(say) … (2.1.1.5)

Here, A1>A2 and, therefore, the range of ρ has been reduced for $\bar{y}_{pr(1)}$ to between more efficient than $\bar{y}$. Clearly $A_3<A_4$. $A_4$ is the lower bound for $\bar{y}_{pr(1)}$ to become more efficient than $\bar{y}_r$ and $\bar{y}$ both if ρ is greater than $A_4$, i.e. for higher correlation between Y and X.

### Relative efficiency of transformed ratio estimators

The relative efficiency of transformed ratio estimators (TRE) is determined against the simple sample mean per unit $\bar{y}$ as

$$E_i = \frac{V(\bar{y})}{MSE(TRE_i)} \times 100$$

Where estimators for $i^{th}$ transformed ratio estimator. Explicitly, they are mentioned below

$$E_1 = \frac{V(\bar{y})}{MSE(y_{lr})} \times 100 \qquad E_6 = \frac{V(\bar{y})}{MSE(y_{sk})} \times 100 \qquad E_{11} = \frac{V(\bar{y})}{MSE(y_{pr(1)})} \times 100$$

$$E_2 = \frac{V(\bar{y})}{MSE(y_r)} \times 100 \qquad E_7 = \frac{V(\bar{y})}{MSE(y_{us(1)})} \times 100$$

$$E_3 = \frac{V(\bar{y})}{MSE(y_{md})} \times 100 \qquad E_8 = \frac{V(\bar{y})}{MSE(y_{us(2)})} \times 100$$

$$E_4 = \frac{V(\bar{y})}{MSE(y_{sd})} \times 100 \qquad E_9 = \frac{V(\bar{y})}{MSE(y_{ms(1)})} \times 100$$

$$E_5 = \frac{V(\bar{y})}{MSE(y_{rr})} \times 100 \qquad E_{10} = \frac{V(\bar{y})}{MSE(y_{ms(2)})} \times 100$$

These relative efficiencies have been worked out for some real populations which are described in Table 1.

**Table.1** Description of populations

| Population No. | Size of the population (N) | Reference | Variable $\bar{Y}$ | Variable $\bar{X}$ |
|---|---|---|---|---|
| 1 | 33 | Cochran, *Sampling techniques*, Third Edition, 2005, pp- 34 | Food Cost | Income |
| 2 | 49 | Cochran, *Sampling techniques*, Third Edition, 2005, pp - 152 | Large U.S. Cities in (1000's) 1930 | Large U.S. Cities in (1000's) 1920 |
| 3 | 89 | Sukhatme and Sukhatme, *Sampling theory of surveys with application*, 1970, pp -256 | Area Under Wheat (Acres) | Number of villages |
| 4 | 25 | Sukhatme and Sukhatme, *Sampling theory of surveys with application*, 1970, pp -51 | Area Under Rice (Acres) | Total Cultivated Area of the Village |
| 5 | 34 | Sukhatme and Sukhatme, *Sampling theory of surveys with application*, 1970, pp -185 | Area Under Wheat in 1936 | Total Cultivated Area in 1931 |
| 6 | 34 | Sukhatme and Sukhatme, *Sampling theory of surveys with application*, 1970, pp -185 | Area Under Wheat in 1937 | Area Under Wheat in 1936 |

**Table.2** Relative efficiency (in %) of the different estimators as compared to simple sample mean $(\bar{y})$ in the different population

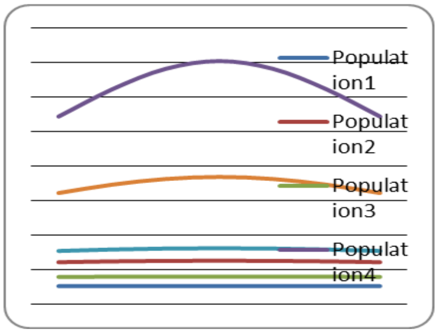
| Estimators | Population | | | | | |
|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** |
| $\bar{y}_{lr}^{*}$ | 106.79 | 253.99 | 162.30 | 1410.99 | 326.13 | 739.40 |
| $\bar{y}_{r}$ | 104.49 | 231.00 | 142.64 | 1339.92 | 325.25 | 700.74 |
| $\bar{y}_{sd}$ | 104.52 | 232.97 | 156.58 | 1338.57 | 325.21 | 703.71 |
| $\bar{y}_{rr}^{*}$ | 106.79 | 253.99 | 162.30 | 1410.99 | 326.13 | 739.40 |
| $\bar{y}_{md}^{*}$ | 106.79 | 253.99 | 162.30 | 1410.99 | 326.13 | 739.40 |
| $\bar{y}_{sk}$ | 104.84 | 243.28 | 152.48 | 1330.06 | 325.06 | 712.59 |
| $\bar{y}_{us(1)}$ | 104.50 | 231.27 | 147.33 | 1339.62 | 325.23 | 701.66 |
| $\bar{y}_{us(2)}$ | 106.12 | 243.16 | 140.83 | 1323.97 | 324.94 | 715.74 |
| $\bar{y}_{ms(1)}$ | 106.67 | 234.80 | 160.59 | 1020.33 | 311.84 | 717.91 |
| $\bar{y}_{ms(2)}$ | 106.05 | 129.84 | 129.87 | 166.18 | 152.66 | 168.05 |
| $\bar{y}_{pr(1)}^{*}$ | 106.79 | 253.99 | 162.30 | 1410.99 | 326.13 | 739.40 |

*At actual values of α, α/β and $\theta' = k$, the relative efficiencies of $\bar{y}_{pr(1)}$, $\bar{y}_{md}$ and $\bar{y}_{rr}$ respectively, are similar to that of regression estimator as their MSE(s) at the respective actual values become equal to the MSE of the regression estimators.

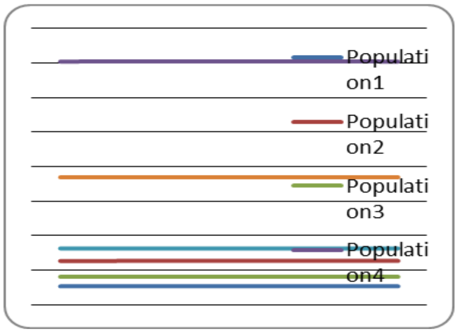
**Table.3** Robustness of estimators in terms of their Relative Efficiency as compared to simple sample mean $(\bar{y})$

| S.No. | Estimator | Parameter | Relative Efficiency (%) of different population | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Value | P1 | Value | P2 | Value | P3 | Value | P4 | Value | P5 | Value | P6 |
| 1. | Reddy Estimator $(\bar{y}_{rr})$ | -15% | 0.54 | 106.63 | 0.68 | 245.48 | 0.58 | 160.05 | 0.91 | 1089.59 | 0.88 | 310.34 | 0.78 | 646.40 |
| | | -10% | 0.57 | 106.72 | 0.72 | 250.13 | 0.61 | 161.29 | 0.96 | 1247.45 | 0.93 | 318.92 | 0.82 | 694.96 |
| | | -5% | 0.61 | 106.77 | 0.76 | 253.01 | 0.65 | 162.04 | 1.01 | 1366.22 | 0.98 | 324.29 | 0.87 | 727.77 |
| | | k Value | 0.64 | 106.79 | 0.80 | 253.99 | 0.68 | 162.30 | 1.07 | 1410.99 | 1.04 | 326.13 | 0.92 | 739.40 |
| | | 5% | 0.67 | 106.77 | 0.84 | 253.01 | 0.71 | 162.04 | 1.12 | 1366.22 | 1.09 | 324.29 | 0.96 | 727.77 |
| | | 10% | 0.70 | 106.72 | 0.88 | 250.13 | 0.75 | 161.29 | 1.17 | 1247.45 | 1.14 | 318.92 | 1.01 | 694.96 |
| | | 15% | 0.73 | 106.63 | 0.92 | 245.48 | 0.78 | 160.05 | 1.23 | 1089.59 | 1.19 | 310.34 | 1.05 | 646.40 |
| 2. | Proposed Estimator $(\bar{y}_{pr(1)})$ | -15% | 8.47 | 106.74 | 24.16 | 253.42 | 295.26 | 161.79 | -42.49 | 1409.31 | -6.65 | 326.11 | 14.55 | 738.48 |
| | | -10% | 9.02 | 106.77 | 25.73 | 253.76 | 314.36 | 162.09 | -45.24 | 1410.32 | -7.09 | 326.12 | 15.49 | 739.03 |
| | | -5% | 9.47 | 106.78 | 27.00 | 253.92 | 329.99 | 162.24 | -47.48 | 1410.81 | -7.44 | 326.13 | 16.26 | 739.30 |
| | | a Value | 9.97 | 106.79 | 28.43 | 253.99 | 347.36 | 162.30 | -49.98 | 1410.99 | -7.83 | 326.13 | 17.12 | 739.40 |
| | | 5% | 10.47 | 106.78 | 29.85 | 253.92 | 364.73 | 162.24 | -52.48 | 1410.81 | -8.22 | 326.13 | 17.97 | 739.30 |
| | | 10% | 10.97 | 106.77 | 31.27 | 253.73 | 382.10 | 162.07 | -54.98 | 1410.25 | -8.61 | 326.12 | 18.83 | 738.99 |
| | | 15% | 11.47 | 106.74 | 32.69 | 253.42 | 399.47 | 161.79 | -57.48 | 1409.31 | -9.00 | 326.11 | 19.69 | 738.48 |
| 3. | Mohanty and Das Estimator $(\bar{y}_{md})$ | -15% | 35.09 | 106.77 | 22.29 | 253.60 | 1.33 | 162.04 | -50.14 | 1409.11 | -22.48 | 326.11 | 17.24 | 738.61 |
| | | -10% | 37.15 | 106.78 | 23.60 | 253.82 | 1.41 | 162.19 | -53.09 | 1410.15 | -23.80 | 326.12 | 18.26 | 739.05 |
| | | -5% | 39.22 | 106.79 | 24.91 | 253.95 | 1.49 | 162.27 | -56.04 | 1410.78 | -25.13 | 326.13 | 19.27 | 739.31 |
| | | a/b Value | 41.28 | 106.79 | 26.22 | 253.99 | 1.56 | 162.30 | -58.99 | 1410.99 | -26.45 | 326.13 | 20.29 | 739.40 |
| | | 5% | 43.35 | 106.79 | 27.53 | 253.95 | 1.64 | 162.27 | -61.94 | 1410.78 | -27.77 | 326.13 | 21.30 | 739.31 |
| | | 10% | 45.41 | 106.78 | 28.84 | 253.83 | 1.72 | 162.20 | -64.89 | 1410.13 | -29.09 | 326.12 | 22.32 | 739.06 |
| | | 15% | 47.47 | 106.77 | 30.15 | 253.65 | 1.80 | 162.08 | -67.84 | 1409.04 | -30.42 | 326.11 | 23.33 | 738.65 |

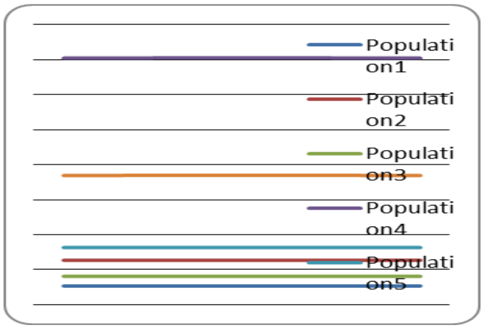
**Note:** $P_i$= population (i= 1, 2,…,6)




(V.N.Reddy)Figure 1




(Mohanty and Das)Figure 2




(Proposed Estimator)Figure 3

**Table.4** The estimator compared with simple sample mean per unit is:

| | | |
|---|---|---|
| 1. | $\bar{y}_{lr} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x})$ | (linear regression estimator) |
| 2. | $\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X}$ | (Usual ratio estimator) |
| 3. | $\bar{y}_{sd} = \bar{y}\frac{\bar{X}+C_x}{\bar{x}+C_x}$ | (Sisodia and Dwevedi,1981) |
| 4. | $\hat{Y}_{rr} = \frac{\hat{Y}X}{\hat{X}-\theta(\hat{X}-X)}$ | (V. N. Reddy,1974) |
| 5. | $\bar{y}_{md} = \frac{\bar{y}}{\left(\bar{x}+\frac{a}{b}\right)}\left(\bar{X}+\frac{a}{b}\right)$ | ( Mohanty and Das, 1971) |
| 6. | $\bar{y}_{SK} = \bar{Y}\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}$ | (Singh and Kakran, 1993) |
| 7. | $\bar{y}_{US(1)} = \bar{y}\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x}$ | (Upadhyaya and Singh, 1999) |
| 8 | $\bar{y}_{US_{(2)}} = \bar{y}\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)}$ | (Upadhyaya and Singh, 1999) |
| 9 | $\bar{y}_{ms(1)} = \frac{\bar{y}}{\bar{z}}\bar{Z}$ | (Mohanty and Sahoo, 1995) |
| 10 | $\bar{y}_{ms(2)} = \frac{\bar{y}}{\bar{u}}\bar{U}$ | (Mohanty and Sahoo, 1995) |

The various parameters of the populations described in the Table 1 have been computed and are presented in the Table 2.
It can be seen from the Table 2 that the relative efficiencies of $\bar{y}_{pr(1)}$, $\bar{y}_{md}$ and $\bar{y}_{rr}$ are exactly similar to that of regression estimators for all the populations. It is due to theoretical fact that at actual values of α, α/β and θ=k respectively the estimators MSE is identical to MSE of the regression estimator. It is obvious that proposed estimator is at par or better than most of the earlier estimators in all the populations. This transformation is unique in the sense that it is obtained by transforming study variate.

## Robustness of some transformed ratio estimators

It can be observed from the table 2 that MSE (s) $\bar{y}_{rr}$ (Reddy, 1974), $\bar{y}_{md}$ (1971) and $\bar{y}_{pr(1)}$ (the proposed transformed ratio estimator) are exactly equal to MSE of $\bar{y}_{lr}$, the usual regression estimator. This has happened because theoretically the MSE (s) of these estimators are equal to the MSE of regression estimator at actual values of α, α/β and $\theta' = k$ , respectively. Therefore, an attempt has been made to examine the robustness of these estimators in terms of loss in precision when there is devotion from the actual values of α, α/β and $\theta' = k$ in this section.

Here α is Y- intercept to regression efficiencies of these estimators have been worked with deviation of ±5%, ±10% and ±15% are presented in the Table 3 for all the populations under study. It can be observed from the table that the losses in efficiencies are quite marginal (practically non-significant) up to the ±10% deviation in the actual values for all the populations. It can also be observed from the table that the proposed estimator $\bar{y}_{pr(1)}$ is more consistent as compared to $\bar{y}_{rr}$ and $\bar{y}_{md}$ from almost all the population. Thus, if the prior values of α, α/β, and k is little away from their actual position, the $\bar{y}_{pr(1)}$ still performs better than other transformed ratio estimators undertaken for the present study.

It was observed on comparison of estimators with sample mean that estimators $\bar{y}_{pr(1)}$, $\bar{y}_{md}$ and $\bar{y}_{rr}$ are exactly similar to that of regression estimators for all the populations. It is due to theoretical fact that at actual values of α, α/β and θ=k respectively the estimators MSE is identical to MSE of the regression estimator. Due to this fact, the robustness of these estimators was examined to observe loss in precision when there is deviation from the actual values of α, α/β and $\theta' = k$

Deviation of $\pm 5\%$, $\pm 10\%$ and $\pm 15\%$ were taken and presented in the Table 4 for all the populations under study. It can be observed from the table that the losses in efficiencies are quite marginal (practically non-significant) up to the $\pm 10\%$ deviation in the actual values for all the populations. It can also be observed from the table that the proposed estimator $\bar{y}_{pr(1)}$ is more consistent as compared to $\bar{y}_{rr}$ and $\bar{y}_{md}$ for almost all the population. Thus, if the prior values of α, α/β, and k are little away from their actual position, the $\bar{y}_{pr(1)}$ still performs better than other transformed ratio estimators undertaken for the present study.

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